

**AI-FARABI KAZAKH NATIONAL UNIVERSITY**  
**FACULTY MECHANICS AND MATHEMATICS**  
**Educational program on specialty «050603-Mechanics»**

**Approved**

at the meeting of Academic Council  
of the faculty of Mechanics and Mathematics

Protocol №\_\_ from «\_\_»\_\_\_\_\_ 2015  
Dean of the Faculty \_\_\_\_\_ Bektemesov M.A.

**NUMERICAL METHODS FOR GASDYNAMICAL PROBLEMS**

**SYLLABUS**

2-nd year master students «050603-Mechanics»,  
Fall semester, 3 credits

**Lecturer/Labs Teacher: Yerzhan Belyayev, Doctor PhD**

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Activities under the program of the course is set in the form of lectures and practical tasks. Practical fastening of the lecture materials is carried out in a laboratory studies and IWMT (independent work of a master student with a teacher) in accordance with the schedule and the program. Tasks for IWM (independent work of a master student) and verification of IWM is carried out by lecturer. Midterm exams takes Labs teacher.

**Aim of the course.** To teach students the basic and modern computational technics in Gas Dynamics, to teach them to understand the basic equations and to solve these equations using numerical procedures. The purpose of discipline is familiarize students with the basic numerical approaches, which are covered with practical examples. Concept of the course is based on the book “Computational Gasdynamics (CGD)” by Culbert B. Laney. As a result of studying the course, students should know the basic system of equations for compressible flows, numerical methods of solution these equations, pros and cons of that numerical methods.

**Objectives of the course.** To teach students to build mathematical models, to understand the basics of gas dynamics such as compressibility, shock waves, Mach number, diffusion etc. and instill skills to recognize specifically behavior of that effects during solving internal and external flow problems. Gas dynamic flow are one of the main types of flow encountered in many practical problems. As it is well known, experimental studies of gas dynamic processes, in particular at high Mach number are costly and unwieldy. So today it is popular to use methods of CFD (Computational fluid dynamics). The main problem in modeling this kind of flow is the numerical solution of nonlinear convective term in the system of equations for compressible viscous (inviscid) gas. This system of equations for a compressible gas describes processes such as the interaction of shock wave structure, shear flows, wake after the body etc. Moreover, for a more detailed and accurate prediction of flow with discontinuities is used schemes

of high order of accuracy (TVD, ENO, WENO), which are based on the physical conditions of the problem, for example, entropy condition. In order to understand all of these issues it is need a basic theoretical knowledge of the program of gas dynamics. For the correct numerical implementation of governing equations with appropriate boundary conditions it is need to know the principals of CGD, which will be taught in this course.

**Learning outcomes.** Necessary knowledge in the basics of gas dynamics and skills to numerically solve the problems of compressible flows.

General competence:

- instrumental – the ability to assess the methodological approaches to carry out their critical analysis;
- interpersonal – ability to independently develop and deepen their knowledge and acquire new skills in a professional manner; knowledge of a foreign language in an amount sufficient to communicate freely in arbitrary topics;
- system – the ability to plan the steps of solving professional problems and implement them in time; demonstrate independence and original approach to problem solving, the ability to justify and make decisions.

**Subject specific competences:** owning a deep fundamental practical knowledge in the CGD, modern calculation technics (TVD, ENO, WENO) for problems of gas dynamics.

**Prerequisites:** “Fluid Mechanics”, “Continuum Mechanics”, “Differential Equations”, “Mathematical Physics”, “Thermodynamics”, “CFD”.

**Post requisites:** “Gas Dynamics”, “Thermodynamics”, “Acoustics”, “CFD”.

### STRUCTURE AND CONTENT OF THE SUBJECT

Week	Title of the theme	Hour	Grade
1	<b>Lecture 1.</b> Compressible Flow – Some History and Introductory Thoughts <b>Lab.1.</b> Modern Compressible Flow. <b>IWM 1.</b> Aerodynamic forces on a body.	2  1	14
2	<b>Lecture 2.</b> Integral forms of the conservation equations for inviscid flows. <b>Lab.2.</b> Continuity and momentum equations. <b>IWM 2.</b> Energy equation.	2  1	14
3	<b>Lecture 3.</b> One-Dimensional Flow <b>Lab.3.</b> Speed of Sound and Mach Number. <b>IWM 3.</b> Hugoniot Equation.	2  1	14
4	<b>Lecture 4.</b> Conservation and other basic principles. <b>Lab.4.</b> The CFL condition. <b>IWM 4.</b> Upwind and adaptive stencils.	2  1	14
5	<b>Lecture 5.</b> Artificial viscosity. <b>Lab.5.</b> Linear stability. <b>IWM 5.</b> Nonlinear stability.	2  1	14
6	<b>Lecture 6.</b> Basic numerical methods for scalar conservation laws. <b>Lab.6.</b> Godunov’s first-order upwind method. <b>IWM 6.</b> Roe’s first-order upwind method.	2  1	14
7	<b>Lecture 7.</b> Beam-warming second-order upwind method.	2	

	<b>Lab.7.</b> Harten's first-order upwind method. <b>IWM 7.</b> Test problem.	1	16
	<b>1<sup>st</sup> control test</b>	1	<b>100</b>
	<b>Midterm exam</b>	1	<b>100</b>
<b>8</b>	<b>Lecture 8.</b> Basic numerical methods for the Euler equations. <b>Lab.8.</b> Flux approach. <b>IWM 8.</b> Wave approach.	2 1	12
<b>9</b>	<b>Lecture 9.</b> Boundary treatments. <b>Lab.9.</b> Second and higher order accurate methods. <b>IWM 9.</b> Test problem.	2 1	12
<b>10</b>	<b>Lecture 10.</b> Flux averaging I: flux-limited methods. <b>Lab.10.</b> Flux-limited TVD. <b>IWM 10.</b> Second and third order accurate methods.	2 1	12
<b>11</b>	<b>Lecture 11.</b> Flux averaging II: flux-corrected methods (FCT). <b>Lab.11.</b> Harten's FCT-TVD. <b>IWM 11.</b> Shu-Osher method ENO.	2 1	12
<b>12</b>	<b>Lecture 12.</b> Flux averaging III: self-adjusting hybrid methods. <b>Lab.12.</b> Harten's self-adjusting hybrid methods. <b>IWM 12.</b> Jameson's self-adjusting hybrid methods.	2 1	12
<b>13</b>	<b>Lecture 13.</b> Solution averaging: reconstruction-evolution methods. <b>Lab.13.</b> Van Leer's reconstruction evolution method MUSCL. <b>IWM 13.</b> Anderson –Thomas- Van Leer reconstruction evolution methods (TVD/MUSCL)	2 1	12
<b>14</b>	<b>Lecture 14.</b> Harten-Osher reconstruction-evolution method UNO <b>Lab.14.</b> Harten-Engquist-Osher-Chakravarthy reconstruction-evolution method ENO <b>IWM 14.</b> Third-order accurate temporal evolution for scalar conservation laws.	2 1	12
<b>15</b>	<b>Lecture 15.</b> WENO scheme. <b>Lab.15.</b> A brief introduction to multidimensions. <b>IWM 15.</b> Prepare a presentation.	2 1	16
	<b>2<sup>nd</sup> control test</b>	1	100
	<b>Exam</b>		<b>100</b>
	<b>TOTAL</b>		<b>(1CT+2CT)/2*0.6 +0.1*MT+0.3*EX AM</b>

## LIST OF LITERATURE

### Main:

1. John D. Anderson, Jr. *Modern Compressible Flow // Second Edition. International Edition 1990.*
2. Culbert B. Laney *Computational Gasdynamics // Cambridge University Press 2007, P. 613.*
3. T. J. Chung *Computational Fluid Dynamics // Cambridge University Press 2002, P. 1012.*
4. К. Флетчер *Вычислительные методы в динамике жидкостей // Москва «Мир» 1991, Том 1,2. 502 с.*

### Additional:

1. C. Hirsch *Numerical Computation of Internal and External Flows: The Fundamentals of Computational Fluid Dynamics // First published by John Wiley & Sons, Ltd, Second Edition 2007, P. 680.*
2. Д. Андерсон, Дж. Таннехил, Р. Плетчер *Вычислительная гидромеханика и теплообмен // Москва «Мир» 1990, Том 1, 2. 726 с.*

## GUIDLINES

All the assignments must be completed until due date. Students, who could not earn 50% out of 100% during first or second midterm and final, will be able to work off during an additional term. Late assignment is not accepted except for extenuating circumstances (e.g. field trip, hospitalization). Student, who failed to meet all kinds of work, is not allowed for passing an exam. In addition, the assessment takes into account the activity and attendance of students during class.

Be tolerant and respect other people's opinions. The objections should be formulated in a correct manner. Plagiarism and other forms of cheating are not allowed. Cheating is not accepted during independent work of student (IWS), midterm and final exam, copying solved problems from others, passing the exam to another student are not allowed also. Student convicted of falsifying any information about the course, any unauthorized upload to the "Intranet" using cheat sheets, will be graded with a final grade «F». For advice on the implementation of IWS, submitting and defending, as well as additional information on the studied material and all the other issues that arose upon studying the course, contact the instructor during his office hours.

Letter grade	Numerical equivalency	% (percentage)	Grading in a traditional way
A	4,0	95-100	Excellent
A-	3,67	90-94	
B+	3,33	85-89	Good
B	3,0	80-84	
B-	2,67	75-79	
C+	2,33	70-74	Satisfactory
C	2,0	65-69	
C-	1,67	60-64	

D+	1,33	55-59	
D-	1,0	50-54	
F	0	0-49	Unsatisfactory
I (Incomplete)	-	-	«The course is incomplete» (this isn't taken into account when calculating the <i>GPA</i> )
P (Pass)	-	-	«Passed» (this isn't taken into account when calculating the <i>GPA</i> )
NP (No Pass)	-	-	«Not passed» (this isn't taken into account when calculating the <i>GPA</i> )
W (Withdrawal)	-	-	«the course is withdrawn» (this isn't taken into account when calculating the <i>GPA</i> )
AW (Academic Withdrawal)			Withdrawn because of academic issues (this isn't taken into account when calculating the <i>GPA</i> )
AU (Audit)	-	-	«Audit» (this isn't taken into account when calculating the <i>GPA</i> )
Att.		30-60 50-100	Attested
Not att.		0-29 0-49	Not attested
R (Retake)	-	-	Retaking the course

*Considered in department meeting*  
*Protocol № \_\_ from «\_\_» \_\_\_\_\_*

**Head of the department of Mechanics** \_\_\_\_\_

Z. Rakisheva

**Lecturer** \_\_\_\_\_

Ye. Belyayev